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Technical Note

Temperature fields of the extruded pipe under conditions of co-current cooling

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Abstract

This article examines the temperature fields of the thick wall pipe while cooling it under the process of extrusion. We solved the Fourier–Kirchhoff equation by Fourier method in the form of an infinite row and with the help of Bessel's functions. The equations were transformed into dimensionless forms and a solution of during heat we got as the function of Biot's and Fourier's number, dimensionless inner radius and thermal capacitance ratio of contact phases.

Keywords: Extrusion; Non-stationary temperature field; Thermal stress

1. Introduction

The uses of plastic pipes in different fields of industry have been on the rise. Also, they have been widely used in the transportation of drinking water and sewage water, and in the transportation of energy sources like gas and oil. Their main advantage is their corrosion resistance and resistance towards aggressive media. However, among definite disadvantages rank their lesser stiffness and limitation for lower temperatures. In order to guarantee the sufficiently precise geometry of pipes one must know their thermal profile already at their production. Non-stationary temperature fields cause [2–4] stress [1] already in the phase of production, as well as residual stresses in the product itself. In this publication a solution of thermal fields will be offered at some simplification of thermal process conditions.

2. Formulation of the problem

In the formulation of the problem we introduce the following simplifying conditions. At the beginning of coordinate system the processed extruded bar enters into the calculation of an ideal cylindrical shape thick wall pipe of outer radius R and inner radius R_1, Fig. 1. The bar material at the beginning is uniformly heated and it has an initial temperature T_{s0}. Around the bar there is a cylindrical space created by a perfectly isolated larger pipe, where the co-current cooling (heating) medium enters with an initial temperature T_{f0} and it is in direct contact with the extruded bar. The motion of the bar is steady and according to the moving piston effect it predetermines the fluid/gas flow in the space. Inside the solid phase we do not consider heat sources. During the solution we assume the thermo-mechanical material properties of the bar material and the gas $/c_f$, c_s, λ_f, $\lambda_s/$ to be constant, that is independent of the temperature. The coefficient of heat transfer between the bar wall and the gas remains constant as well. The heat due to radiation is included in the coefficient of heat transfer α. The mass flow of the gas M_f and the bar material M_s does not vary with time.

3. Mathematical description of the problem

Considering the aforementioned simplifications of the Fourier–Kirchhoff equation of heat conduction in

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Nomenclature

a	coefficient of temperature diffusivity [$m^2 s^{-1}$]
c	specific heat [$J kg^{-1} K^{-1}$]
C	integration constant
E	Young modulus [MPa]
G	modulus of elasticity in shear [MPa]
J	Bessel's function first kind
Y	Bessel's function second kind
m	thermal capacitance ratio of contact phases [–]
M	mass flow [$kg s^{-1}$]
R	outer radius of cylinder [m]
R_1	inner radius of cylinder [m]
t	time [s]
T	temperature [K]
α	coefficient of heat transfer [$Wm^{-2} K^{-1}$]
ρ	dimensionless radial coordinate [–]
λ	heat conductivity [$Wm^{-1} K^{-1}$]
Θ	dimensionless temperature [–]

Subscripts

f	fluid phase
s	solid phase
0	initial value
p	variable value on the out surface
c	calorimetric
0	0-th order
1	1-th order
i	inner

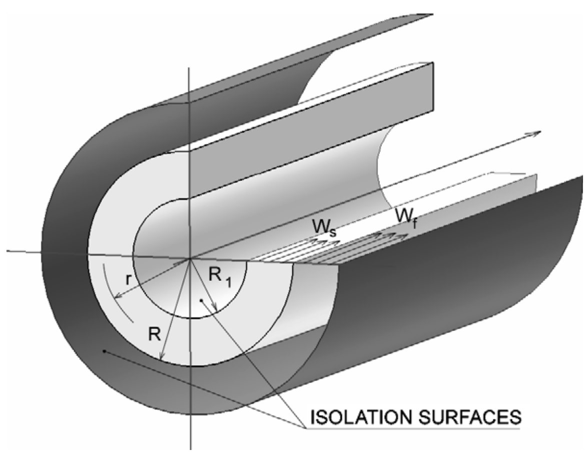

Fig. 1. Solid and liquid phase flux direction.

cylindrical coordinate, it can be transformed into the following term:

$$\frac{\partial T}{\partial t} = a\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right). \tag{1}$$

The initial and boundary conditions are the following: the temperature of the bar as well as of the gas at the entry is constant

$$t = 0, T_s = T_{s0}, T_f = T_{f0}. \tag{2}$$

Under the assumption that the inner surface of the bar is perfectly insulated, the following term is valid:

$$\left[\frac{\partial T_s}{\partial r}\right]_{r=R_1} = 0. \tag{3}$$

The condition of temperature exchange on the boundary of both contact phases is described by this equation:

$$\alpha[T_f - (T_s)_{r=R}] = -\lambda_s\left[\frac{\partial T_s}{\partial r}\right]_{r=R}, \tag{4}$$

while the following term is valid: $T_s|_{r=R} = T_{sp}$. If we introduce average calorimetric temperature in the solid phase

$$T_{sc} = \frac{1}{(1-\rho_1^2)}\int_{\rho_1}^{1} 2\frac{r}{R}T_s\frac{dr}{R} = \frac{2}{(1-\rho_1^2)}\int_{\rho_1}^{1}\rho T_s\,d\rho, \tag{5}$$

then we can find the relation between the temperature of the gas and the temperature in the solid phase in the following heat balance law

$$M_s c_s(T_{sc} - T_{s0}) = M_f c_f(T_{f0} - T_f). \tag{6}$$

4. Solving the problem

We introduce the following dimensionless variables:

$Bi = \frac{\alpha R}{\lambda_s}$	Biot number,
$Fo = \frac{at}{R^2}$	Fourier number
$m = \frac{M_s c_s}{M_f c_f}$	thermal capacitance ratio of the contact phases
$\rho = \frac{r}{R}$	dimensionless coordinate, $\rho \in (\rho_1; 1)$,
$\Theta_s = \frac{T_s - T_{s0}}{T_{f0} - T_{s0}}$	relative temperature difference of the solid phase,
$\Theta_{sc} = \frac{T_{sc} - T_{s0}}{T_{f0} - T_{s0}}$	average calorimetric relative temperature difference
$\Theta_{sp} = \frac{T_{sc} - T_{s0}}{T_{f0} - T_{s0}}$	surface relative temperature difference,
$\Theta_f = \frac{T_f - T_{s0}}{T_{f0} - T_{s0}}$	relative temperature difference of the fluid phase

(7)

The conduction equation can be combined as follows:

$$\frac{\partial \Theta_s}{\partial Fo} = \frac{\partial^2 \Theta_s}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial \Theta_s}{\partial \rho} \tag{8}$$

and the Fourier heat conduction equation on the outside surface

$$1 - m\Theta_{sc} - \Theta_{sp} = -\frac{1}{Bi}\left[\frac{\partial \Theta_s}{\partial \rho}\right]_{\rho=1}, \tag{9}$$

respectively, boundary conditions on the inner surface (zero heat flow)

$$\left[\frac{\partial \Theta_{\mathrm{s}}}{\partial \rho}\right]_{\rho=\rho_1} = 0. \tag{10}$$

Initial conditions of the problem will be the following:

$$Fo = 0; \quad \Theta_{\mathrm{s}} = \Theta_{\mathrm{sp}} = 0, \quad \Theta_{\mathrm{sc}} = 0,$$

equation the condition of temperature exchange on the boundary will have the following form:

$$\Theta_{\mathrm{f}} = 1 - m\Theta_{\mathrm{sc}}, \tag{11}$$

and in the end the relative average calorimetric temperature difference in the solid phase

$$\Theta_{\mathrm{sc}} = \frac{2}{(1-\rho_1^2)} \int_{\rho_1}^{1} \rho \Theta_{\mathrm{s}}\,\mathrm{d}\rho. \tag{12}$$

Utilizing the Fourier method after substitutions, the temperature field of the infinite thick wall pipe can be found in terms of an infinite series as a function of dimensionless time Fo, coordinate ρ, inner dimensionless radius ρ_1, temperature capacitance ratio m and Biot number Bi

$$\Theta_{\mathrm{s}} = \frac{1}{1+m} - \sum_{i=1}^{\infty} D_{\mathrm{i}} \mathrm{e}^{-k_i^2 Fo} (Y_1(k_i\rho_1) J_0(k_i\rho) - J_1(k_i\rho_1) Y_0(k_i\rho)), \tag{13}$$

or after launching the substitution

$$V_0(k_i\rho) = Y_1(k_i\rho_1) J_0(k_i\rho) - J_1(k_i\rho_1) Y_0(k_i\rho) \tag{14}$$

$$V_1(k_i\rho) = Y_1(k_i\rho_1) J_1(k_i\rho) - J_1(k_i\rho_1) Y_1(k_i\rho) \tag{15}$$

$$\Theta_{\mathrm{s}} = \frac{1}{1+m} - \sum_{i=1}^{\infty} D_i \mathrm{e}^{-k_i^2 Fo} V_0(k_i\rho). \tag{16}$$

From the Eq. (12) we can determine dimensionless medium calorimetric temperature of the solid phase

$$\Theta_{\mathrm{sc}} = \frac{1}{1+m} + \frac{2}{(1-\rho_1^2)} \times \sum_{i=1}^{\infty} D_i \mathrm{e}^{-k_i^2 Fo} \left[\frac{V_1(k_i) - \rho_1 V_1(k_i\rho_1)}{k_i}\right], \tag{17}$$

and from the Eq. (11) the temperature of the liquid phase

$$\Theta_{\mathrm{f}} = \frac{1}{1+m} - \frac{2m}{(1-\rho_1^2)} \times \sum_{i=1}^{\infty} D_i \mathrm{e}^{-k_i^2 Fo} \left[\frac{V_1(k_i) - \rho_1 V_1(k_i\rho_1)}{k_i}\right]. \tag{18}$$

The values $V_1(k_{\mathrm{i}})$ and $V_1(k_{\mathrm{i}}\rho_1)$ in the Eqs. (17) and (18) can be obtained from the Eqs. (14) and (15) by means of substituting the coordinates of the surfaces.

The constants k_i (which are dependent on the Biot number, dimensionless radius ρ_1 and the thermal capacitance ratio of contact phases) can be determined according to the following transcendental equation

$$0 = -k_i^2 V_1(k_i) + \frac{2mBi}{(1-\rho_1^2)} V_1(k_i) + Bik_i V_0(k_i). \tag{19}$$

The constants D_i we get from the boundary conditions

$$D_i = \frac{2k_i(1-\rho_1^2)(V_1(k_i) - \rho_1 V_1(k_i\rho_1))}{4m(V_1(k_i) - \rho_1 V_1(k_i\rho_1))^2 + (1-\rho_1^2)k_i^2\left[V_0^2(k_i) + V_1^2(k_i) - \rho_1^2(V_0^2(k_i\rho_1) + V_1^2(k_i\rho_1))\right]} \tag{20}$$

The shape of temperature surface in dependence on the dimensionless time Fo and the radius ρ in the thick wall pipe for subsequent values of the dimensionless quantities

$m = 1$

$Bi = 2$

$\rho_1 = 0.5$

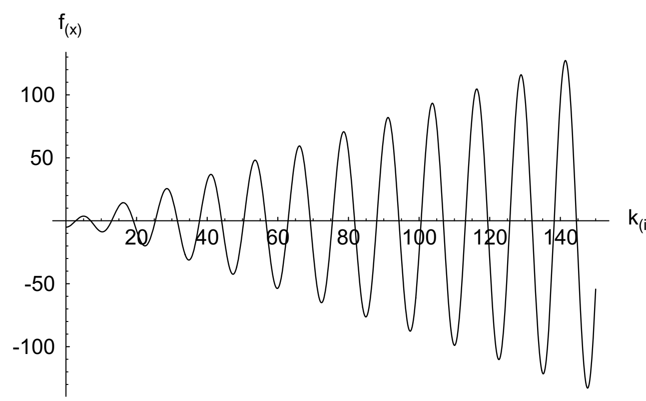

Fig. 2. Graphical representation of the transcendental equation.

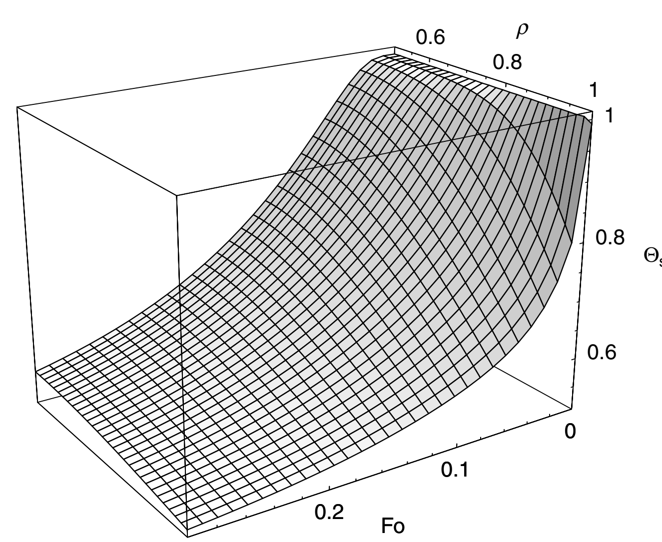

Fig. 3. Temperature distribution over the pipe wall.

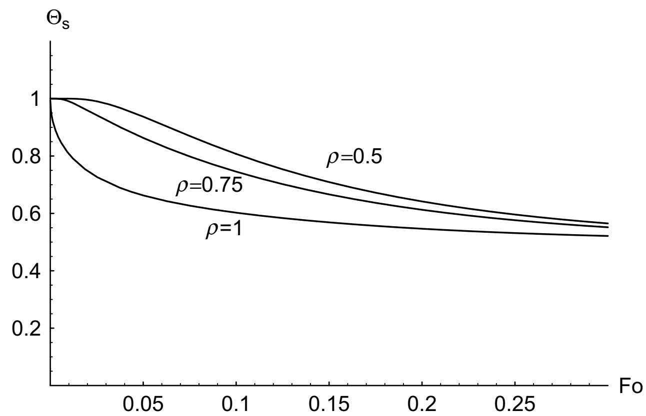

Fig. 4. Temperature distribution on the surfaces and mean radius $\rho = 0.75$.

is represented by means of software Mathematica in the Fig. 2.

The shape of curve the dimensionless temperature on the outer and inner surface (for previous conditions) in the dependence the dimensionless time is represented in the Fig. 3. Fig. 4 shows the dimensionless temperature of surfaces corresponding to the inner radius, outer radius and $\rho = 0.75$ versus Fourier number curve.

5. Closure

The presented paper solves the non-stationary temperature field of the infinite thick wall pipe at the co-current contact with liquid medium. The derived analytical solution shows us the dimensionless temperature dependence of the pipe wall on the dimensionless coordinate ρ, the dimensionless inner radius ρ_1, Fourier number Fo, Biot number Bi and temperature capacitance ratio of both phases m.

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References

[1] P. Élesztös, Thermal stresses at the extrusion of an infinite cylinder, Int. J. Mech. Solids (IJM&S) 1 (1) (2006).

[2] P. Élesztös, Non-stationary temperature field of infinite cylinder at co-current contact with liquid medium, Period. Politech. Ser. Mech. Eng. 48 (2) (2004), Budapest.

[3] A. Bejan, A.D. Kraus, Heat Transfer Handbook, John Wiley & Sons, Inc., Hoboken, NJ, 2003.

[4] P. Élesztös, Non-stationary temperature field of an infinite plate at co-current contact with fluids, in: Proceedings of the CO-MA-TECH'98 Conference, October 1998, Trnava, Slovakia (in Slovak).